Oberseminar

Lie superalgebras

WS 2017/18

Termin: **Mo. 15:30 – 17:00 Uhr (Raum 48-438)** Beginn: 23. 10. 2017

23.10.17	—	**Introduction + talk distribution** (16:15-17:00 Uhr)
30.10.17	Gunter Malle:	**Finite-dimensional complex Lie algebras**
06.11.17	Melvin Dauter:	**Lie superalgebras: Basic definitions**
13.11.17	William Wong:	**The universal enveloping algebras / linear representations**
20.11.17	Robin Ammon:	**Simple Lie superalgebras**
27.11.17	Benjamin Sambale:	**The classical simple Lie superalgebras I**
04.12.17	Jayati Kaushik:	**Borel subalgebras**
07.12.17	Emil Rotilio:	**Lie superalgebras in Physics**
11.12.17	Patrick Wegener:	**Dynkin-Kac diagrams**
08.01.18	Niamh Farrell:	**The classical simple Lie superalgebras II**
15.01.18	Daniel Schaefer:	**Classification of the classical simple Lie superalgebras**
22.01.18	Ruwen Hollenbach:	**Cartan type Lie superalgebras**
29.01.18	Alessandro Paolini:	**Classification of the simple Lie superalgebras I**
05.02.18	Caroline Lassueur:	**Classification of the simple Lie superalgebras II**

Literatur

[FSS00] L. Frappat, A. Sciarrino, and P. Sorba, *Dictionary on Lie algebras and superalgebras*, Academic Press, Inc., San Diego, CA, 2000.

[Kac77a] V. G. Kac, *Lie superalgebras*, Advances in Math. **26** (1977), 8–96.

[Kac77b] ———, *A sketch of Lie superalgebra theory*, Comm. Math. Phys. **53** (1977), 31–64.

[Mus12] I. M. Musson, *Lie superalgebras and enveloping algebras*, Graduate Studies in Mathematics, vol. 131, American Mathematical Society, Providence, RI, 2012.

[Sch79] M. Scheunert, *The theory of Lie superalgebras: An introduction*, Lecture Notes in Mathematics, vol. 716, Springer, Berlin, 1979.

Interessierte Hörer sowie weitere Vortragende sind herzlich willkommen!

Talk Description (tentative)

Below is a short description of the content of each talk. We will essentially follow the books by Musson [Mus12] and Scheunert [Sch79] and Kac's article [Kac77a]. The survey article [Kac77b] is also helpful in order to get the gist of the topic. The dictionary [FSS00] can also bring additional information.

Talk length: 90 minutes

30.10.2017 – TALK I. Finite-dimensional complex Lie algebras
The aim of this talk is to provide us with a short recap of the important concepts of the theory of finite-dimensional complex Lie algebras: nilpotent, soluble, semisimple, simple Lie algebras, the Killing form, root systems, the classification of finite-dimensional simple Lie algebras over $\mathbb{C}$.

06.11.2017 – TALK II. Lie superalgebras: Basic definitions
- Introduce notational conventions for the whole semester ([Sch79, Chap. 0, §1]).
- Do a short recap on graded algebraic structures ([Sch79, Chap. 0, §2] and [Kac77a, §1.1.1]).
- Define: Lie superalgebras, morphisms thereof, the general linear Lie superalgebra, the supertrace, superderivations, the adjoint representation ([Sch79, Chap.I, §1] and [Kac77a, §1.1.2, §1.1.4, §1.1.5]).

13.11.2017 – TALK III. The universal enveloping algebras / linear representations
- Introduce the enveloping algebra of a Lie superalgebra and state the PBW Theorem and its corollaries ([Mus12, §6.1], [Sch79, Chap. I, §2.1-2.3], [Kac77a, §1.1.3]).
- Introduce basic definitions/operations on linear representations and modules ([Sch79, Chap. I, §3.1-3.2])

20.11.2017 – TALK IV. Simple Lie superalgebras
- Define: the commutator subalgebra, abelian, nilpotent, soluble and simple Lie superalgebras.
- Representations and modules: complete if necessary with definitions and elementary properties not introduced in Talks II and III irreducibilty, Schur's Lemma,...)
- Cover [Mus12, §1.2] on simple Lie superalgebras. See also [Sch79, Chap. II, §2].

27.11.2017– TALK V. The classical simple Lie superalgebras I
The aim of this talk is to describe the different families of classical simple Lie superalgebras: Start by introducing root space decompositions ([Mus12, Chap. 2, §2.1]). Then define:
- the Lie superalgebras of type $A(m, n)$ ([Mus12, Chap. 2, §2.2]);
- the orthosymplectic Lie superalgebras ([Mus12, Chap. 2, §2.3]);
- the strange Lie superalgebras of type $\mathfrak{p}(n)$ and $\mathfrak{q}(n)$ ([Mus12, Chap. II, §2.4]).

Introduce the Killing form ([Mus12, Chap. 2, §2.6]).

04.12.2017 – TALK VI. Borel subalgebras
Cover [Mus12, §3.1-3.3]:
- Cartan subalgebras and Borel-Penkov-Serganova subalgebras ([Mus12, §3.2]);
- Flags, shuffles and Borel subalgebras ([Mus12, §3.3]).

07.12.2017. Lie superalgebras in Physics

11.+18.12.2017 – TALKS VII+VIII. Dynkin-Kac diagrams I & II
The aim of these talks is to associate a set of simple roots and a so called *Dynkin-Kac diagram* to the Borel subalgebras constructed in Talk VI. The two speakers should agree on how to split ([Mus12, §3.4]). (Talk VIII: cancelled)

08.01.2018 – TALK IX. The classical simple Lie superalgebras II
Give explicit constructions for the exceptional Lie superalgebras
- $D(2,1;\alpha)$,
- $G(3)$, and
- $F(4)$.
([Mus12, Chapter 4])

15.01.2018 – TALK X. Classification of the classical simple Lie superalgebras
The aim of this talk is to explain the main steps of the classification of the classical simple Lie superalgebras. Use [Kac77a, §2.2-2.5] and [Sch79, Chap. II, §5].

22.01.2018 – TALK XI. Cartan type Lie superalgebras
Describe the construction of the simpe Lie superalgebras of Cartan type
- $W(n)$,
- $S(n)$,
- $\widetilde{S}(n)$, and
- $H(n)$.
([Sch79, Chap. II, §6] and [Kac77a, §3]).

29.01.2018 – TALK XII & XIII. Classification of the simple Lie superalgebras I & II
[Sch79, Chap. II, §8]