Oberseminar

Perverse equivalences and applications

WS 2018/19

Organized by Alessandro Paolini

Termin: **Mo. 15:30 – 17:00 Uhr (Raum 48-438)** Beginn: 22. 10. 2018

22.10.18	—	**Introduction + talk distribution**
29.10.18	Bernhard Böhmler:	**Projective and injective modules**
05.11.18	Arezoo Aslizadeh:	**Categories and functors**
12.11.18	Ruwen Hollenbach:	**Resolutions and complexes**
19.11.18	Emil Rotilio:	**Mapping cones and triangulated categories**
26.11.18	Alessandro Paolini:	**Derived categories**
03.12.18	Madeleine Whybrow:	**Derived functors**
10.12.18	Olivier Dudas:	**Derived and perverse equivalences**
17.12.18	Patrick Wegener:	**Examples of derived equivalences**
07.01.19	Caroline Lassueur:	**Symmetric algebras**
14.01.19	Niamh Farrell:	**Perverse equivalences and rationality**
21.01.19	Gunter Malle:	**Brauer tree algebras I**
28.01.19	Alessandro Paolini:	**Brauer tree algebras II**
Apr/May 19	David Craven (TBC):	**Guest talk**

Literatur

[Alperin] J. L. Alperin, *Local representation theory*, Cambridge University Press, 1993.

[ChKe] J. Chuang and R. Kessar, *On perverse equivalences and rationality*, preprint, arXiv:1811.01000 (2018).

[ChRo] J. Chuang and R. Rouquier, *Perverse equivalences*, preprint, `http://www.math.ucla.edu/~rouquier/papers/perverse.pdf` (2017).

[Dud1] O. Dudas, *Lectures in modular Deligne-Lusztig theory*, EMS Series of Lectures in Mathematics, Vol. 29 (2018), 107–177. Available at `https://arxiv.org/pdf/1705.08234.pdf`.

[Dud2] O. Dudas, *Perverse equivalences*, handout.

[DS] L. Dreyfus-Schmidt, *Équivalences perverses: le cas des algèbres symétriques*, handout. Available at `https://webusers.imj-prg.fr/~olivier.dudas/src/dreyfus-schmidt-GTperv-04-13.pdf`.

[KZ] S. König and A. Zimmermann, *Derived equivalences for group rings*, Lecture Notes in Mathematics, Springer, 2006.

[Rotman] J. J. Rotman, *Advanced modern algebra*, 2nd ed., Providence, RI: American Mathematical Society (AMS), 2010.

Interessierte Hörer sowie weitere Vortragende sind herzlich willkommen!

Talk Description (tentative)

Perverse equivalences are a particular case of derived equivalences, whose construction can be determined explicitly and algorithmically. They naturally arise in the context of representation theory of finite groups, for instance in the determination of the behavior of group algebras and Iwahori-Hecke algebras and of the structure of blocks with cyclic defect groups.

The structure of the Oberseminar is as follows. In the first half of the course, we review results on modules and categories ([Rotman, Chapter 7]), and we get to the definition and first properties of perverse equivalences after surveying the basics on triangulated categories, derived categories and derived functors ([KZ, Chapter 2] and [Dud1, Chapter 1]). In the second half of the course, we investigate some applications of derived equivalences to path algebras ([KZ, Chapter 3]), and of perverse equivalences to symmetric algebras ([DS] and [Dud2, Section III]) and to Brauer tree algebras ([Alperin, Chapter 5] and [ChRo, §8.4]).

The course organizer is indebted to Dr. Olivier Dudas for useful discussions about the structure of the course.

Talk length: 90 minutes

Note: Speakers should decide whether there is time for any proofs that are not explicitly mentioned in the program. In affirmative case, they should select to present the ones that they find to be the most interesting.

22.10.18 – TALK 0. Introduction and talk distribution

During this shorter lecture, given by the course organizer, the structure of the Oberseminar will be outlined in more detail with a full list of lectures, and each participant to the course will be assigned one topic to present on a certain Monday slot 15:30-17:00.

[Suggestion: the slots for the topics are allocated on a first come, first served basis. If you are passionate about one of the topics and/or figure out it is easy to present, then proposing to give the corresponding lecture is a win-win!]

29.10.18 – TALK I. Projective and injective modules

- Free modules and universal properties. Rank. [Rotman, Def. 471, Prop. 7.49, Def. 472].
- (Short) exact sequences. (Non-)splitting. [Rotman, Def. 435–437, Prop. 7.22, Eg. 437].
- Projective modules. Equivalent definitions. Non-free projectives. Schaunel's Lemma. [Rotman, Def. 474, merge Prop. 7.53, (Prop. 7.54 with proof) and Thm. 7.56; Eg. 7.57, Prop. 7.60].
- Injective modules. Equivalent definitions. Relation with free and projective modules. [Rotman, Def. 480, merge Prop. 7.63 and (Prop. 7.64 with proof), Cor. 7.65, Eg. 7.74]

05.11.18 – TALK II. Categories and functors
- Categories: objects, morphisms. Examples. Equivalences. [Rotman, Def. 442, Eg. 7.25 (i)–(iv), Def. 444].
- Functors. Contravariance. [Rotman, Def. 461, Eg. 7.40 (i),(ii),(v) (brief), Def. 463].
- Hom functor. Left/right exactness. [Rotman, Thm. 7.44, Eg. 7.45 (brief), Def. 468, Thm. 7.46, Eg. 7.47, Def. 469 (brief), Def. 470]

12.11.18 – TALK III. Resolutions and complexes
- Generators and relations. Free, projective and injective resolutions. [Rotman, Prop. 7.51, Rmk. and Def. 813, Prop. 10.32 with proof, Def. 814, Prop. 10.33]
- Complexes. Differentials. [Rotman, Def. 815, Eg. 10.34 (i), (vii), (viii)]
- Chain maps. n-cycles and n-boundaries. n-th homology (group). [Rotman, Def.s 817, Def. 818, Prop. 10.37, Def. 819]
- Grothendieck groups [Dud1, Recall Def. of abelian categories; §4.1 first paragraph]

19.11.18 – TALK IV. Mapping cones and triangulated categories
- Null-homotopy. Homotopy category and s.e.s. [Dud1, §1.2+Rmk.s, Def. 1.3, Ex. 1.4].
- Uniqueness of projective resolutions in $Ho(A-Mod)$. Quasi-isomorphisms. Mapping cones and morphisms of complexes. [Dud1, Prop. 1.5, Ex. 1.6, Def. qis + Def. 1.8, Prop. 1.9].
- Triangle. Morphisms of triangles. Distinguished triangles. [Dud1, The rest of §1.2].
- Axiomatic definition of triangulated categories. Relation of octahedral axiom with third isomorphism theorem. [KZ, Thm. 2.3.1].
Try to maintain the notation as uniform as possible.

26.11.18 – TALK V. Derived categories
- f qis iff $M(f)$ has zero cohomology. Multiplicative and null- systems. [KZ, Rmk.s before Def. 2.5.2, Def. 2.5.2, Def. 2.5.3, Prop. 2.5.1]
- Recall triangulated categories. Derived categories: definition. Qis and mapping cones. [KZ, Prop. 2.5.3, Rmk.s before and afterwards]
- Relation between a ring A and the derived category of $A-Mod$. Qis which are not isomorphisms. [Dud1, Prop. 1.12, Rmk. afterwards]
- $A-Proj$ and $A-Inj$, relations with $Ho(A-Mod)$, natural isomorphisms of abelian groups Hom. [Dud1, Def. before Lem. 1.13, Prop. 1.15, Thm. 1.16]
Try to maintain the notation as uniform as possible. Try to give ideas of important proofs. If time allows, state and give ideas of the proof of Lem. 1.13.

03.12.18 – TALK VI. Derived functors
- Left and right derived functors. First definitions of Ext and Tor [Dud1, Thm. 1.16, Eg. 1.18 (a)-(b)].
- Axiomatic definition of Ext. Calculation in one case. (Split) extensions. Ext^1 and projectives. [Rotman, Thm. 10.45, Eg. 10.84, Def. 855, Prop. 10.85, Cor. 10.86].
- Compare [Rotman, Def. 832, Def. 834-835] with [Dud1, pages 10–11] to give further insight on derived functors.
- Tor and tor are well-defined. Properties. Relations with tensors and ses. [Rotman,

Def. 836, Cor. 10.51, Thm. 10.56, Cor. 10.57, Eg. 10.98].
Only if time allows: Equivalent extensions. $e(C, A)$ and bijection with Ext [Rotman, Def. 856, Thm. 10.89 with no proof but the idea].

10.12.18 – TALK VII. Derived and perverse equivalences
We first give some examples of derived equivalences. We then give motivation for the introduction of perverse equivalences in the theory. We define them and we illustrate the first properties. We state (and possibly sketch) the proof of Rickard's Morita theorem.

17.12.18 – TALK VIII. Examples of derived equivalences
- Review of Auslander-Reiten quivers. [KZ, Rmk. in §2.11 before §2.11.1].
- Introductory discussion (brief). Tilting modules. (Mutually inverse equivalences on subcategories.) Some equivalence involving the derived category of A and its construction. [KZ, §3.1, recall projective dimension, Def. 3.1.1(, Thm. 3.1.1), Thm. 3.1.2].
- Tilting complexes. Some equivalences of triangulated categories. [KZ, Def. 3.2.1 and remarks before it, Thm. 3.2.1]
- Main example: three path-algebras which are derived equivalent. [KZ, Eg. §§2.11.1–2.11.3, second part of §3.2]

07.01.19 – TALK IX. Symmetric algebras
The talk is a selection of the statements and proofs in the following list.
- Definition, examples, properties. Lemma Omnibus. [DS, page 1]
- Serre functors. properties. Characterising some derived equivalences. Reductions to the symmetric case. Bijections of simples. [DS, pages 2 and 3]
- Non increasing perversity. The theorem of Chuang and Rouquier on non-increasing perversities. [DS, pages 4–5], [Dud2, pages 15–16].
- Particular case $\pi(i) = 1$, $i \in \{0, 1\}$. (Pre-)images of simples via perversity. [Dud2, pages 17–18 with proof].
- Perverse equivalences as compositions of elementary equivalences. Brauer trees. Images of simples. [Dud2, page 19].
More details on the topic can be found in [ChRo, Section 5].

14.01.19 – TALK X. Perverse equivalences and rationality
The goal of this talk is to survey the recent article [ChKe] on the fact that perverse equivalences preserve rationality. Applications towards long-standing conjectures in representation theory of finite groups will also be presented.

21.01.19 – TALK XI. Brauer tree algebras I
The aim of the lecture is to provide a summary of [Alperin, Chapter 5] containing the main definitions and the most important ideas on blocks with cyclic defect group by defining and studying Brauer tree algebras. If time allows, some notions from Talk XII and applications to perverse equivalences can be already introduced.

28.01.19 – TALK XII. Brauer tree algebras II
- Recall Brauer tree algebras and blocks with cyclic defect group. Recall Rickard's construction of a certain derived equivalence. [ChRo, beginning of §8.4].
- Perverse equivalences of Brauer tree algebras. Applications to Brauer correspondence. Combinatorics and structure transformation of Brauer trees. [ChRo, Thm. 8.8 and Cor. 8.9, give ideas; page 40].
- $End(T_A(I))$ as a Brauer tree algebra. Exceptional vertex and its multiplicity. Brief comments on previous results. [ChRo, Prop. 8.10, few important ideas on the proof; page 43; ideas from Lemma 4.20 and 5.5].
- Example of perverse equivalence with a star Brauer tree algebra. [ChRo, page 44; extract needed ideas from proof of Prop. 8.10].
If time allows, one can go deeper into the proof of [ChRo, Prop. 8.10].

April/May 2019 – TALK XIII. Guest talk (TBA)
An external guest will give a lecture on a topic on perverse equivalences and their applications to representation theory.